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Revisiting Noether symmetry approach in F(R)-tachyon Model

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Abstract

Recently, some authors have made a falsifiable claim that Noether gauge symmetry for $F(R)$ theory of gravity coupled to a tachyon field enforces gauge to vanish and leads to $F(R) \propto R^2$, with a tachyon potential $V(\phi) \propto \phi^{-4}$. Here, we show that the analysis is completely wrong since the conserved current does not satisfy the field equations. Earlier, it has been shown that $F(R)$ theory of gravity in vacuum or in matter dominated era admits trivial Noether symmetry for $F(R) \propto R^{\frac{3}{2}}$, because $z = a^2$, a being the scale factor, becomes cyclic for $F(R) \propto R^{\frac{3}{2}}$. Further, it has already been noticed that $F(R)$ does not admit Noether symmetry when coupled to a scalar sector, minimally or non-minimally. Finally, here we show that Noether symmetry is obscure even when $F(R)$ is coupled to a tachyon field. However, Noether symmetry for R^2 gravity coupled to a scalar taking an auxiliary variable Q different from R exists in the literature and it is not recovered from $F(R)$ gravity. Since, canonization of a general $F(R)$ theory of gravity is only possible treating R as the auxiliary variable, so we conclude that in general Noether symmetry of $F(R)$ theory of gravity is obscure.

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1 Introduction

Interest in $F(R)$ theory of gravity has increased predominantly in the recent years (see [1] for a recent review and references therein). It is therefore important to apply Noether symmetry as a selection rule to find a form of $F(R)$. Canonical formulation of a general $F(R)$ theory of gravity is only possible treating R as an auxiliary variable, provided $F''(R) \neq 0$ (here, dash represents derivative with respect to R). In the process, the gravitational Lagrangian turns out to be a point Lagrangian, and one can demand Noether symmetry to find a suitable form of $F(R)$. Following this technique, several authors [2] found $F(R) \propto R^{\frac{3}{2}}$, in the Robertson-Walker metric in vacuum or considering pressureless dust in the $F(R)$ theory of gravity. Although such a form of $F(R)$ shows accelerating expansion from early deceleration in the matter dominated era, however, early deceleration tracks as $a \propto t^{\frac{1}{2}}$ instead of usual $t^{\frac{2}{3}}$, which creates problem in explaining structure formation. Further, it yields $a \propto t^{\frac{4}{3}}$ in the radiation dominated era instead of usual $t^{\frac{1}{2}}$, which contradicts CMBR data. Thus, simply $R^{\frac{3}{2}}$, in the absence of a linear term in the action, does not worth explaining presently available cosmological data [3]. It has been also noticed that [3] instead of the scale factor a, if one would have started with $h_{ij} = z = a^2$, then z becomes cyclic both in vacuum and matter dominated era. Thus Noether symmetry obtained in the process is trivial. Situation could have been improved if Noether symmetry allows linear term R in the action, but it does not. To obtain a better form of $F(R)$, scalar field has been incorporated both minimally and non-minimally, but Noether symmetry remains obscure [4]. This is astonishing, since on the contrary, Noether symmetry exists for a scalar-tensor theory of gravity in the presence of R^2 [5]. But such result is not recovered from $F(R)$ theory of gravity. This contradiction puts up doubt in treating R as an auxiliary variable for canonical formulation of $F(R)$ theory of gravity. the reason is, an auxiliary variable Q different from R was taken to canonize R^2 theory of gravity. Let us discuss the issue in some detail.

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In the quantum domain observable depends on the choice of momentum, while momentum is different for different auxiliary variable. The canonical formulation of R^2 gravity in view of the auxiliary variable Q, leads to a Schrödinger like quantum dynamics, with a hermitian effective action and a straightforward probability interpretation [6]. From metric variation principle it is known that R^2 theory of gravity must be supplemented by a boundary term $\Sigma = 4\beta\int\ ({}^4R)K\sqrt{h}\ d^3x$, where symbols have their usual meaning. It was shown [6] that to obtain Schrödinger like quantum equation as mentioned, it is required first to express the action in terms of the first fundamental form h_{ij} and then to split the above boundary term into $\Sigma = \sigma_1 + \sigma_2$ where, $\sigma_1 = 4\beta\int\ ({}^3R)K\sqrt{h}\ d^3x$ and $\sigma_2 = 4\beta\int\ ({}^4R -{}^3 R)K\sqrt{h}\ d^3x$. Canonical programme then follows by eliminating the available total derivative term from the action, which gets cancelled with the boundary term σ_1, and the auxiliary variable, $Q = \frac{\partial A}{\partial \dot{h}_{ij}}$, as suggested by Horowitz [7] should be introduced thereafter. Thus Q is different from R, in the R^2 theory of gravity. It is important to note that such technique is not possible to follow for a general $F(R)$ theory of gravity and this may be the reason why it could not reproduce Noether symmetry for $F(R) = R^2$ theory of gravity already available in the literature [5].

Despite the fact that Noether symmetry yields nothing other than $F(R) \propto R^{\frac{3}{2}}$, in vacuum or in matter dominated era, some authors have recently claimed [8] that it might yield arbitrary form of $F(R)$ taking into account a gauge term in the Noether theory. Ridiculously, even after setting the gauge term to zero, it was claimed to be an outcome of gauge Noether symmetry. It has been shown [9] that the result [8] is wrong since neither the Noether equations nor the field equations are satisfied for such symmetry. More recently, the same authors have claimed that [10] Noether symmetry yields $F(R) \propto R^2$ taking tachyon field into account. In the present letter we review the work and show that again the conserved current thus obtained does not satisfy the field equations here too. Further, since the authors [8, 10] have taken time translation into account, it appears that indefinitely large number of such symmetry may be present. This is again not true. The gravitational Hamiltonian is not only conserved but also constrained to vanish and is a part of the field equations. Thus it is unnecessary to consider time translation in this situation and make things complicated.

In a recent article [9] we have shown how gauge term modifies Noether symmetry for a point Lagrangian. In the following section we briefly review this issue to show that the gauge term has to be time independent for a time independent lagrangian, as in the case of gravity. Further, we shall give the form of the vector field and the conserved current taking gauge into account. In section 3, we write down the Noether equations for an action containing $F(R)$ being minimally coupled to a tachyon field. We then review the earlier work [10] to explore gauge Noether symmetry for $F(R)$ theory in the presence of a tachyon field. We finally show that the conserved current does not satisfy the field equations. Next, in the same section, we handle the situation in a straightforward manner to show that Noether symmetry enforces $F(R)$ to vanish. This proves our earlier claim [4] that Noether symmetry for $F(R)$ theory of gravity is obscure in general.

2 Noether symmetry with gauge

Among all the dynamical symmetries, transformations that map solutions of the equations of motion into solutions, one can single out Noether symmetries as the continuous transformations that leave the action invariant - except for boundary terms. In formal language, Noether symmetry states that, for any regular system if there exists a vector field $X^{(1)}$, such that,

$$\left(\pounds_{X^{(1)}} + \frac{d\eta}{dt}\right) L = \left(X^{(1)} + \frac{d\eta}{dt}\right) L = \frac{dB}{dt}, \tag{1}$$

in the presence of a gauge function $B(q_i, t)$, where $X^{(1)}$ is the first prolongation of the vector field X given by,

$$X^{(1)} = X + \sum_i \left[(\dot{\alpha}_i - \dot{\eta}\dot{q}_i)\frac{\partial}{\partial \dot{q}_i}\right] \ \ \&\ \ X = \eta\frac{\partial}{\partial t} + \sum_i \alpha_i \frac{\partial}{\partial \dot{q}_i}, \tag{2}$$

with, $\alpha_i = \alpha_i(q_i, t),\ \eta = \eta(q_i, t)$, then there exists a conserved current,

$$I = \sum_i(\alpha_i - \eta\dot{q}_i)\frac{\partial L(q_i, \dot{q}_i, t)}{\partial \dot{q}_i} + \eta L(q_i, \dot{q}_i, t) - B(q_i, t) = \sum_i \alpha_i p_i - B(q_i, t) - \eta(q_i, t)H(q_i, p_i, t), \tag{3}$$

Let us now try to understand the issue of gauge in terms of the point Lagrangian. Although we are dealing with field theory, however to explore Noether symmetry, we reduce the gravitational Lagrangian to point Lagrangian which does not contain time explicitly. Therefore, to understand the issue of gauge it is sufficient to deal with

point Lagrangian. For a point Lagrangian, Noether symmetry states that under infinitesimal transformations of co-ordinates ($q_i' = q_i + \epsilon\alpha_i(q_i, t)$) and time ($t' = t + \epsilon\eta(q_i, t)$), if the Hamilton's principal function remains unchanged then there exists a conserved current. Invariance of Hamilton's principal function under infinitesimal transformation implies,

$$\int L'(q_i', \dot{q}_i')dt' = \int L(q_i, \dot{q}_i)dt,$$

in the absence of explicit time dependance in the Lagrangian. Now, $L(q_i', \dot{q}_i', t')$ may be generated from $L'(q_i', \dot{q}_i')$ under the introduction of time dependent gauge in the following manner,

$$\int L(q_i', \dot{q}_i', t')dt' = \int L'(q_i', \dot{q}_i')dt' + \epsilon \int \frac{dB'(q_i', t')}{dt'}dt'.$$

From the above two equations it follows that

$$\int L(q_i', \dot{q}_i', t')dt' = \int L(q_i, \dot{q}_i)dt + \epsilon \int \frac{dB'(q_i', t')}{dt'}dt'.$$

Now expanding $L(q_i', \dot{q}_i, t')$ and $B'(q_i', t')$ in Taylor series and under suitable transformations of coordinates, velocities and time one ends up with,

$$\int L(q_i, \dot{q}_i, t)dt - \int L(q_i, \dot{q}_i)dt + \epsilon\frac{d}{dt}\int \left[\eta\left(L(q_i, \dot{q}_i, t) - \sum \dot{q}_i\frac{\partial L(q_i, \dot{q}_i, t)}{\partial \dot{q}_i}\right) + \sum \alpha_i\frac{\partial L(q_i, \dot{q}_i, t)}{\partial \dot{q}_i} - B(q_i, t)\right]dt = 0, \tag{4}$$

retaining only up to first order term. It is apparent that the conserved current (3) may be obtained from (4) only if the first two terms of the above expression (4) get cancelled. This is possible provided the Lagrangian appearing in the very first term is time independent, which enforces a time independent gauge term B, as well. Hence, the very important point to remember about Noether gauge symmetry is that, for Lagrangian which does not contain time explicitly (as in the case of gravity), the gauge should have to be independent of time. Further, the trivial Noether point symmetry is $\frac{\partial}{\partial t}$, for which Noether integral is the Hamiltonian. Now for a system in which the Hamiltonian is conserved, the integral of motion $I_1 = H$, which demands homogeneity of time and so $\eta = 1$. This is the situation one encounters in gravity, where, not only the Hamiltonian is conserved, but it is constrained to vanish. As a result, the field equations contain the Hamiltonian constraint equation and it is useless to consider time translation in the case of gravity. This is the reason why earlier authors in this field never considered time translation. Thus the integral of motion (3) reduces simply to

$$I = \sum_i \alpha_i p_i - B(q_i, t). \tag{5}$$

Essentially if gauge turns out to be zero while solving Noether equations, the integral of motion remains the same as for Noether symmetry without gauge and no new result is expected. This is the fallacy in the works of Hussain et al [8] and Jamil et al [10]. In section 3, we show explicitly that indeed Noether symmetry is obscure for $F(R)$ theory of gravity when coupled to a scalar field.

3 Action, field equations and Noether equation

In spatially flat Robertson-Walker line element

$$ds^2 = -dt^2 + a^2\left[dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2\right], \tag{6}$$

the following Born-Infeld effective 4-dimensional action for a rolling tachyon field ϕ with Lagrangian density $\mathcal{L}(\phi) = V(\phi)\sqrt{1 - \zeta\phi_{,\mu}\phi^{,\mu}}$ being minimally coupled to $F(R)$

$$A = \int d^4x\sqrt{-g}\left[F(R) + V(\phi)\sqrt{1 - \zeta\phi_{,\mu}\phi^{,\mu}}\right], \tag{7}$$

leads to a point lagrangian

$$L = 6a\dot{a}^2F' + 6a^2\dot{a}\dot{R}F'' + a^3(F'R - F) - a^3V(\phi)\sqrt{1-\zeta\dot{\phi}^2}, \tag{8}$$

treating

$$R - 6\left(\frac{\ddot{a}+\dot{a}^2}{a^2}\right) = 0, \tag{9}$$

as a constraint of the theory, and effectively spanning the Lagrangian by a set configuration space variables $(a, R, \phi, \dot{a}, \dot{R}, \dot{\phi},)$. In the above action ζ is treated as the coupling constant required to make the kinetic part of the action dimensionless. For the above point Lagrangian the Noether equation (1) reads,

$$\begin{aligned}
&\alpha\left[6\dot{a}^2F' + 12a\dot{a}\dot{R}F'' + 3a^2(F'R-F) - 3a^2V\sqrt{1-\zeta\dot{\phi}^2}\right] + \beta\left[6a\dot{a}^2F'' + 6a^2\dot{a}\dot{R}F''' + a^3RF''\right] + \gamma[-a^3V_{,\phi}\sqrt{1-\zeta\dot{\phi}^2}]\\
&\quad+\left[(\alpha_{,t} - \dot{a}\eta_{,t}) + \alpha_{,a}\dot{a} + \alpha'\dot{R} + \alpha_{,\phi}\dot{\phi} - \eta_{,a}\dot{a}^2 - \eta'\dot{a}\dot{R} - \eta_{,\phi}\dot{a}\dot{\phi}\right][12a\dot{a}F' + 6a^2\dot{R}F'']\\
&\quad+\left[(\beta_{,t} - \dot{R}\eta_{,t}) + \beta_{,a}\dot{a} + \beta'\dot{R} + \beta_{,\phi}\dot{\phi} - \eta_{,a}\dot{a}\dot{R} - \eta'\dot{R}^2 - \eta_{,\phi}\dot{R}\dot{\phi}\right][6a^2\dot{a}F'']\\
&\quad+\left[(\gamma_{,t} - \dot{\phi}\eta_{,t}) + \gamma_{,a}\dot{a} + \gamma'\dot{R} + \gamma_{,\phi}\dot{\phi} - \eta_{,a}\dot{a}\dot{\phi} - \eta'\dot{R}\dot{\phi} - \eta_{,\phi}\dot{\phi}^2\right]\left[\frac{\zeta a^3V\dot{\phi}}{\sqrt{1-\zeta\dot{\phi}^2}}\right]\\
&+\left[\eta_{,t} + \eta_a\dot{a} + \eta'\dot{R} + \eta_{,\phi}\dot{\phi}\right]\left[6a\dot{a}^2F' + 6a^2\dot{a}\dot{R}F'' + a^3(F'R-F) - a^3V\sqrt{1-\zeta\dot{\phi}^2}\right] = B_{,a}\dot{a} + B'\dot{R} + B_{,\phi}\dot{\phi} + B_{,t},
\end{aligned} \tag{10}$$

where, dash ($'$) represents derivative with respect to R. Note that we have kept both time translation and a time dependent gauge function since it was taken by the earlier work performed by Jamil et al [10]. Now, equating coefficients as usual, we obtain a large over-determinant set of Noether equations for $F''(R) \neq 0$. These are

$$\eta_{,a} = \eta' = \eta_{,\phi} = 0 \tag{11}$$
$$\alpha' = \alpha_{,\phi} = 0 \tag{12}$$
$$\beta_{,\phi} = 0 \tag{13}$$
$$\gamma_{,t} = \gamma' = \gamma_{,a} = 0 \tag{14}$$
$$B_{,\phi} = 0 \tag{15}$$
$$\gamma_{,\phi} - \eta_{,t} = 0 \tag{16}$$
$$6a^2F''\alpha_{,t} = B' \tag{17}$$
$$12aF'\alpha_{,t} + 6a^2F''\beta_{,t} = B_{,a} \tag{18}$$
$$a^2(3\alpha + a\eta_{,t})(F'R - F) + a^3\beta RF'' = B_{,t} \tag{19}$$
$$\alpha F' + a\beta F'' + 2a\alpha_{,a}F' + a^2\beta_{,a}F'' - aF'\eta_{,t} = 0 \tag{20}$$
$$2a\alpha F'' + a^2\beta F''' + a^2F''(\alpha_{,a} + \beta' - \eta_{,t}) = 0 \tag{21}$$
$$3\alpha V + a\gamma V_{,\phi} + aV\eta_{,t} = 0 \tag{22}$$

Before we proceed let us mention that if $V(\phi) = 0$, the action (7) corresponds to pure $F(R)$ theory of gravity, for which nothing other than $F(R) \propto R^{\frac{3}{2}}$ is possible [3]. In what follows, we shall assume $V(\phi) \neq 0$ in equation (22) and so it will never be possible to recover pure $F(R)$ case. Now, in view of equations (11) through (15), it is apparent that $\eta = \eta(t)$, $\alpha = \alpha(t, a)$, $\beta = \beta(t, a, R)$, $\gamma = \gamma(\phi)$ and $B = B(t, a, R)$. Therefore equation (16) dictates that η may be at most linear in t. Thus equation (22) clearly states that α has to be time independent (as γ is) and it should be linear in the scale factor a. Thus taking $\alpha = c_4a$, where c_4 is a constant, it is clear that β has to be independent of t and a in view of equations (20) and (21) and the gauge term B must be independent of R as is apparent from equation (17). Further, since both α and β are time independent, therefore the gauge term further becomes independent of the scale factor a in view of equation (18). Finally, equation (19)

is satisfied provided $B \neq B(t)$. In the process the gauge term B turns out to be a constant and hence plays no role in Noether symmetry. Thus one can set $B = 0$, without loss of generality. In view of the above analysis, we now have,

$$\eta = c_1 t + c_3, \quad \alpha = c_4 a, \quad \beta = \beta(R), \quad \gamma = c_1\phi + c_2 \; \textit{and} \; B = 0, \tag{23}$$

where, c_1, c_2, c_3 and c_4 are constants. Having obtained explicit forms of η, α and γ, we are now left to find the explicit forms of $\beta(R)$, $F(R)$ and the potential $V(\phi)$ from the set of over-determinant equations (19) through (22) which in view of (23) now take the following forms

$$\begin{aligned}
(3c_4 + c_1)(F'R - F) + F''R\beta &= 0 && (24)\\
(3c_4 - c_1)F' + \beta F'' &= 0 && (25)\\
\beta F''' + F''(3c_4 + \beta' - c_1) &= 0 && (26)\\
(3c_4 + c_1)V + (c_1\phi + c_2)V_{,\phi} &= 0 && (27)
\end{aligned}$$

3.1 A quick review of Jamil et al's work

Equation (27) may now be solved for the following form of the potential

$$V = V_{10}(c_1\phi + c_2)^{-\left(\frac{3c_4+c_1}{c_1}\right)} = c_1 V_{10}\left(\phi + \frac{c_2}{c_1}\right)^{-\left(\frac{3c_4+c_1}{c_1}\right)}, \tag{28}$$

V_{10} being a constant. However, the authors [10] claimed the following form of the potential

$$V = V_0(\phi + \phi_0)^{-4}, \tag{29}$$

which is possible only under the choices, $V_0 = c_1 V_{10}$, $\phi_0 = c_2/c_1$ and in particular, $c_1 = c_4$, which as we shall show shortly, create severe problem. One can easily check that under the choice $c_1 = c_4$, equation (26) upon integration yields equation (25), provided the constant of integration is zero. Thus we are now left with two independent equations (24) and (25) to find the form of $F(R)$ and $\beta(R)$. However, equation (25) now reads

$$\beta F'' = -2c_1 F'. \tag{30}$$

Now eliminating β between equations (24) and (30) one ends up with

$$F(R) \propto R^2 \tag{31}$$

in a straightforward manner, and β gets solved as $\beta = -2c_1 R$. At this end the authors obtained two conserved current I_1 and I_2. I_1 is the invariance under time translation, as stated correctly by the authors (here we point out a typographical error in equation (31) of [10] . I_1 should be $I_1 = \tau\Big[L - \big(\dot a\frac{\partial L}{\partial \dot a} + \dot R\frac{\partial L}{\partial \dot R} + \dot\phi\frac{\partial L}{\partial \dot\phi}\big)\Big]$ instead, where τ used in [10] stands for η here). Nevertheless, the third bracketed term is the Hamiltonian, which is constrained to vanish. Thus if $I_1 = 0$ is substituted in I_2 (as already mentioned in section 2), the conserved current is simply,

$$I_2 = \sum \alpha_i p_i = \left[12F_0\dot R + \frac{\zeta V_0\dot\phi}{(\phi+\phi_0)^3\sqrt{1-\zeta\dot\phi^2}}\right]a^3. \tag{32}$$

Conserved current is not an independent equation, rather it is the first integral of certain combination of the field equations. Thus it is essential to check if the above conserved current obtained for $F(R) = F_0 R^2$ and

$V = V_0(\phi+\phi_0)^{-4}$, satisfy the field equations, which was not performed by the authors [10]. Now the corresponding field equations are

$$F_0\left[2\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}+\frac{\ddot{R}}{R}+2\frac{\dot{a}\dot{R}}{aR}-\frac{R}{4}\right]+\frac{V}{4R}\sqrt{1-\zeta\dot{\phi}^2}=0. \tag{33}$$

$$\ddot{\phi}+3\frac{\dot{a}}{a}\dot{\phi}-3\zeta\frac{\dot{a}}{a}\dot{\phi}^3+\frac{V_{,\phi}}{\zeta V}(1-\zeta\dot{\phi}^2)=0. \tag{34}$$

$$H=F_0[12a\dot{a}^2R+12a^2\dot{a}\dot{R}-a^3R^2]+\frac{a^3V(\phi)}{\sqrt{1-\zeta\dot{\phi}^2}}=0. \tag{35}$$

It is not difficult to check that the above conserved current (I_2) satisfies the field equations only under the trivial condition $R=0$ together with the condition $V_0=0$ i.e., for vanishing potential, which leads to inconsistency, since it has been restricted at the beginning. Thus $F(R)\propto R^2$ is not a symmetry of the action under consideration, and the work performed by Jamil et al [10] is completely wrong. One can even look at the consequence in a straightforward manner. It is well-known that for Noether point symmetry $\frac{\partial}{\partial t}$ the Noether integral is the Hamiltonian, i.e., I_1 is the Hamiltonian H. But here, $I_1=\eta H=(c_1t+c_3)H$. Thus unless $\eta=1$, implying $c_1=0$ and $c_3=1$, Hamiltonian is not obtained as the integral of motion. However, since $c_1=c_4$, it can not be set equal to zero at this stage, since it makes $\alpha=\beta=0$ and $\gamma=$ constant, while the form of the potential (28) becomes undefined. Thus the Hamiltonian is never recovered as Noether integral. As mentioned in section 2, that to recover the Hamiltonian constraint equation as an outcome of Noether point symmetry $\frac{\partial}{\partial t}$, one should start with $c_1=0$ a-priori, which we consider in the following subsection.

3.2 No need to consider time translation, so, $\eta=c_3=1$.

It must have been clear by this time why earlier authors did not consider time translation. Likewise, if one starts with $\eta=1$, which implies $c_1=0$ and $c_3=1$, γ turns out to be a constant and may be set as $\gamma=c_2=1$, without loss of generality. thus equations (24) through (27) take the following forms,

$$\begin{aligned}3c_4(F'R-F)+RF''\beta &= 0 \quad (36)\\ 3c_4F'+F''\beta &= 0 \quad (37)\\ \beta F'''+F''(3c_4+\beta') &= 0 \quad (38)\\ 3c_4V+V_{,\phi} &= 0 \quad (39)\end{aligned}$$

Now, just multiplying equation (37) by R and comparing it with equation (36), one can see $F(R)=0$. Thus Noether symmetry remains obscure for $F(R)$ theory of gravity even when it is coupled with a tachyon field.

4 Comments

Earlier attempts in finding Noether symmetry for $F(R)$ theory of gravity in the vacuum and matter dominated era gave $F(R)\propto R^{\frac{3}{2}}$ [2]. Such a form is not suitable to explain presently available cosmological data [3]. Search of a better form of $F(R)$ taking a scalar field minimally or non-minimally into account failed to produce symmetry [4]. Apparently, its not a problem, since not all actions admit symmetry. But then, such a symmetry for $F(R)\propto R^2$ already exists in the literature where an auxiliary variable Q different from R was introduced for the purpose of canonization [5]. Hence the result should have been reproduced from $F(R)$ theory of gravity. Canonization of the action, treating R as the auxiliary variable creates problem in quantum domain [6] and so it appears that this may be the root of trouble.

However, some authors claimed that $F(R)$ in vacuum admits such symmetry for arbitrary power of R, if a gauge term is introduced [8]. Ridiculously, even after setting gauge term to zero, they claimed the result to be an outcome of gauge Noether symmety. It has been pointed out that their result is completely wrong, since neither all Noether equations are satisfied nor the conserved current thus obtained satisfies the field equations [9]. Here again the

authors did the same and claimed that Noether symmetry exists for $F(R) \propto R^2$, but now in the presence of tachyon field [10]. Not only that the conserved current does not satisfy the field equations, but also from their consideration of time translation, it appears that the authors are totally ignorant of the fact that the $(0\,0)$ equation of Einstein is the Hamiltonian in disguise which is not only conserved but also constrained to vanish. Since, Hamiltonian is a part of the field equations, therefore the earlier authors did not consider time translation. It has been shown in section 3.2 that a single line calculation reveals that Noether symmetry is obscure for tachyon field. Since all attempts to find Noether symmetry for $F(R)$ theory of gravity (other than the trivial one, viz., $F(R) \propto R^{\frac{3}{2}}$) so far has failed, we conclude that Noether symmetry is obscure for $F(R)$ gravity in general and so further attempts in the same line, are useless. One has to select some form of $F(R)$ a-priori, find auxiliary variable judiciously for canonization and then should proceed to find such symmetry, as was done for R^2 theory of gravity [5]. On the contrary, we remind that there exists a conserved current in general for $F(R)$ gravity being coupled to a scalar field non-minimally [4, 11, 12]. Such a conserved current is not associated with symmetry of any type and is of help to study cosmological solutions [4, 12].

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